Nuclear Energy

Mid-term exam solutions

19-09-2017

Problem 1. (4 pts)

Radium-226 has a half-life of 1600 years. Calculate the activity of 1 g of ^{226}Ra.

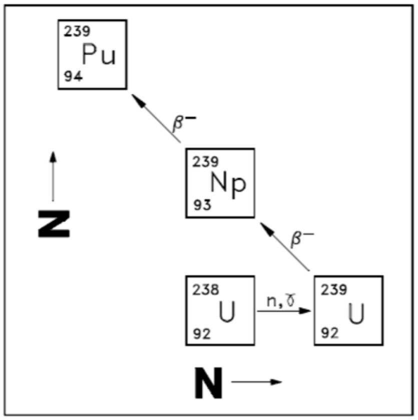
Solution: decay constant $\lambda = \ln 2/T_{1/2} = 0.693/(1600*365*24*3600) = \mathbf{1.37\ 10^{-11}\ s^{-1}}$

$N = N_A/226 = 6.022\ 10^{23}/226 = \mathbf{2.66\ 10^{21}}$

Activity $A = \lambda N = \mathbf{3.65\ 10^{10}}$ **Bq**

Problem 2. (15 pts)

The picture below shows that the capture of one neutron by ^{238}U ($T_{1/2}$ = 4.468 billion years) forms ^{239}U ($T_{1/2}$ = 23.45 min), which in turn decays to ^{239}Np ($T_{1/2}$ = 2.36 days). ^{239}Np then decays to ^{239}Pu ($T_{1/2}$ = 24110 years).



1. Complete the reaction: ^{238}U + n → … → ^{239}Pu

2. The flux of neutrons in a reactor is $\phi = 10^{14}$ cm^{-2} s^{-1} and the radiative capture cross section for thermal neutrons if σ = 2.7 b.

   Calculate the fraction of ^{239}U nuclei that transform into ^{239}Pu in one year (neglect neutron captures on ^{239}U, ^{239}Np and the fission of ^{239}Pu). Assume that the number of ^{238}U nuclei at t = 0 is N_0 and that there are no other nuclei present.

3. If the reactor starts with one tonne of ^{238}U, how much ^{239}Pu is produced?

Solution:

1. $^{238}U + n \rightarrow {}^{239}U^* \rightarrow {}^{239}Np + e^- + \bar{\nu} \rightarrow {}^{239}Pu + e^- + \bar{\nu}$
2. Let's call ^{238}U A, ^{239}U B, ^{239}Np C and ^{239}Pu D.

   The creation of nuclei B is given by $\frac{dN_A}{dt} = -\lambda_A N_A$ and

   $$\lambda_A = \sigma_{capt}\,\phi = 2.7\times10^{-24}\,(cm^2)\times10^{14}\left(cm^{-2}s^{-1}\right) = 2.7\times10^{-10}\,s^{-1}$$

The bateman equations are given by:

$$N_N = N_0(h_A e^{-\lambda_A t} + h_B e^{-\lambda_B t} + \ldots + h_N e^{-\lambda_N t})$$

$$h_A = \frac{\lambda_A}{\lambda_N - \lambda_A}\frac{\lambda_B}{\lambda_B - \lambda_A}\frac{\lambda_C}{\lambda_C - \lambda_A}\cdots\frac{\lambda_M}{\lambda_M - \lambda_A}$$
$$h_B = \frac{\lambda_A}{\lambda_A - \lambda_B}\frac{\lambda_B}{\lambda_N - \lambda_B}\frac{\lambda_C}{\lambda_C - \lambda_B}\cdots\frac{\lambda_M}{\lambda_M - \lambda_B}$$
$$h_N = \frac{\lambda_A}{\lambda_A - \lambda_N}\frac{\lambda_B}{\lambda_B - \lambda_N}\frac{\lambda_C}{\lambda_C - \lambda_N}\cdots\frac{\lambda_M}{\lambda_M - \lambda_N}$$

Since the half-life of ^{239}Pu is much larger than those of ^{239}U and ^{239}Np, we can consider ^{239}Pu as stable. Therefore, $\lambda_D = 0$.

This gives:

$$h_A = -\frac{\lambda_B}{\lambda_B - \lambda_A}\frac{\lambda_C}{\lambda_C - \lambda_A}$$
$$h_B = -\frac{\lambda_A}{\lambda_A - \lambda_B}\frac{\lambda_C}{\lambda_C - \lambda_B}$$
$$h_C = -\frac{\lambda_A}{\lambda_A - \lambda_C}\frac{\lambda_B}{\lambda_B - \lambda_C}$$
$$h_D = 1$$

$\lambda_A = 2.7\ 10^{-10}\ s^{-1}$, $\lambda_B = 4.93\ 10^{-4}\ s^{-1}$, $\lambda_C = 3.4\ 10^{-6}\ s^{-1}$ → $\lambda_A << \lambda_C << \lambda_B$

We can then simplify the above equations further:

$$h_A = -1$$
$$h_B = \frac{\lambda_A}{\lambda_B}\frac{\lambda_C}{\lambda_C - \lambda_B} = -\frac{\lambda_A \lambda_C}{\lambda_B^2}$$
$$h_C = \frac{\lambda_A}{\lambda_C}\frac{\lambda_B}{\lambda_B - \lambda_C} = \frac{\lambda_A}{\lambda_C}$$
$$h_D = 1$$

We can now write:

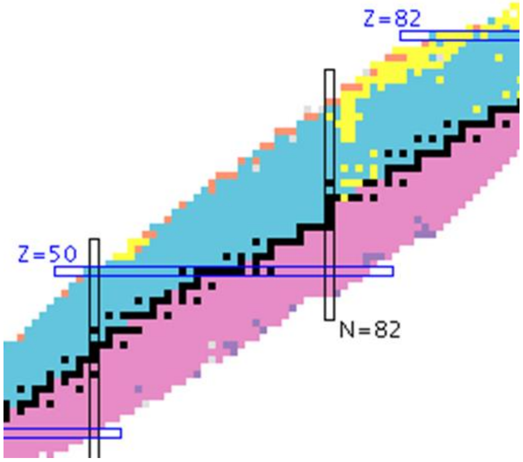
$$N_D = N_A(0)\,(1 - e^{-\lambda_A t} - \frac{\lambda_A \lambda_C}{\lambda_B^2} e^{-\lambda_B t} + \frac{\lambda_A}{\lambda_C} e^{-\lambda_C t})$$

We can now calculate $N_D/N_A(0)$ for t = 1 year: $N_D/N_A(0) = 0.008469 \sim 0.0085$ or 0.85%

3. Starting with 1000 kg of ^{238}U, 8.5 kg of ^{239}Pu were produced.

Problem 3. (6 pts)

The portion of nuclear chart below shows stable nuclei in black and radioactive nuclei in purple/pink and in blue.



1. By what types of decay mode do you expect the blue nuclei to decay and why?
2. Same question for the purple/pink nuclei.

Solution:

1. Blue nuclei have an excess of protons and will likely decay by β^+ and/or by EC
2. Pink nuclei have an excess of neutrons and decay by β^-.

Problem 4. (10 pts)

Consider the following reaction:

n + ^{235}U → ^{236}U* → ^{139}Ba + ^{95}Kr + 2n

Calculate the Q-value of the reaction and the excitation energy of ^{236}U.

M(^{235}U) = 235.0439 u
M(^{236}U) = 236.0455 u
M(^{139}Ba) = 138.9088 u
M(^{95}Kr) = 94.9398 u
M(neutron) = 1.00866 u

Solution:
Q (^{236}U → ^{139}Ba + ^{95}Kr + 2n) = 167.4 MeV
Q(^{235}U + n → ^{139}Ba + ^{95}Kr + 2n) = 173.9 MeV
E(^{236}U*) = 173.9 – 167.4 = 6.5 MeV
Or E(^{236}U*) can be calculated from the Q-value of the n + ^{235}U → ^{236}U* reaction.